RELATION & FUNCTIONS

1. Prove that the relation R on the set N × N defined by (a, b) R (c, d) ⇒ a + d = b + c ∀ (a, b), (c, d) ∈ N × N is an equivalance relation.

2. Consider the set N × N, the set of all ordired pairs of natural numbers. Let R be the relation in N × N which is defined by (a, b) R (c, d) if and only if ad = bc. Prove that R is an equivalence relation.

3. Prove that the relation R on set Z of all integers defined by (a, b) ∈ R ⇔ (a – b) is divisible by 5 is an equivalence relation on Z.

4. Show that the function f : R → R given by f (x) = ax + b, a, b ∈ r, a ≠ o is a bijection.

5. Show that the function $f : R - \{-1\} \to R - \{1\}$ given that $f(x) = \frac{x-3}{x+1}$ is a bi jective function.

6. Let $f : R_+ \to [-5, \infty]$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible and $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

7. Let A be the set of all real numbers except – 1

   Let an operation * be defined on A as a * b = a + b + ab ∀ a, b ∈ A

   Prove that (i) A is closed under the given operation

   (ii) * is commutative as well as associative

   (iii) the number O is the identity element

   (iv) every element a ∈ A has $\frac{-a}{1+a}$ as inverse

8. Let $f : R - \left\{\frac{7}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ be defined as $f(x) = \frac{3x+4}{5x-7}$ and $g : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{7}{5}\right\}$, be defined as $g(x) = \frac{7x+4}{5x-3}$

   Show that fog = I_A and gof = I_B where I_A and I_B are idnetity functions on A and B respectively.

INVERSE TRIGONOMETRIC FUNCTIONS

1. Find the value of $\tan^{-1} 1 + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$ $\left(\text{Ans. } \frac{\pi}{4} + \frac{2\pi}{3} + \frac{\pi}{6} = \frac{13\pi}{12}\right)$

2. Find the value of $\sec^2\left(\tan^{-1} 2\right) + \operatorname{cosec}^2\left(\cot^{-1} 3\right)$ (Ans. 15)

3. Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{31}{17}\right)$

4. Prove that $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

5. Prove that $2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$

6. Prove that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

7. Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$

8. Prove that $\cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right] = \frac{x}{2}, \quad x \in \left]0, \frac{\pi}{4}\right[$

9. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(x^2\right)$

10. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ Prove that $x^2 + y^2 + z^2 + 2xyz = 1$

11. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ Prove that $x + y + z = xyz$

12. (a) Solve : $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$ $\left(\text{Ans. } x = \frac{1}{6}\right)$

    (b) Solve : $2\tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$ $\left(\text{Ans. } x = \frac{\pi}{4}\right)$

13. Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ $\left(\text{Ans. } \pm\frac{1}{\sqrt{2}}\right)$

14. Write $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ in the simplest form.

15. Write $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ in the simplest form.

MATRIX

1. Find X and Y if $2x + y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}$ & $x - 2y = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

2. If $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} X = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$, find X (Ans. $X = [-1 \quad 2 \quad 1]$

3. Find x such that $[1 \quad x \quad 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$ (Ans. $x = -14$ or -2)

4. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then Prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix} \forall n \in N$

5. If $A = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}$ & $B = [3 \quad 1 \quad -2]$ verify that $(AB)' = B'A'$

6. Express the Matrix $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric and skew symmetric Matrix.

7. Using elementary row transformation find the inverse of Matrix

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ Ans. $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

8. Show that $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ satisfies the equation $x^2 + 4x - 42 = 0$. Hence find A^{-1}

Ans. $\frac{1}{42} \begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$

9. If $A = \begin{bmatrix} 2 & 1 & 8 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ show that $A\,(\text{adj}A) = (\text{adj}\,A)\,A = |A|\,I_3$

10. Using Matrix method, solve the following system of linear equations :

$$x + 2y - 3z = -4$$
$$2x + 3y + 2z = 2$$
$$3x - 3y - 4z = 11$$

(Ans. $x = 3$, $y = -2$ & $z = 1$

* List of Activities :

1. To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ is an equivalence relation

2. To draw the graph of $\sin^{-1} x$, using the graph of $\sin x$ and demonstrate the concept of mirror reflection (about the line $y = x$)

Activity 2

Objective

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ is an equivalence relation.

Material Required

A piece of plywood, some pieces of wire (8), plywood, nails, white paper, glue.

Method of Construction

Take a piece of plywood of convenient size and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig. 2.

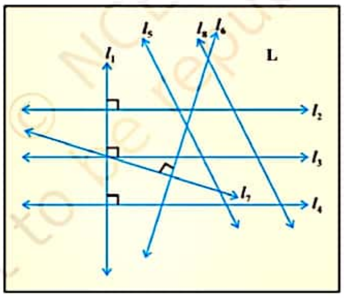

Fig. 2

Demonstration

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4 (see Fig. 2).

3. l_6 is perpendicular to l_7.
4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.
5. $(l_2, l_3), (l_3, l_4), (l_5, l_8), \in R$

Observation

1. In Fig. 2, every line is parallel to itself. So the relation $R = \{(l, m) : l \parallel m\}$ .... reflexive relation (is/is not)
2. In Fig. 2, observe that $l_2 \parallel l_3$. Is l_3 ... l_2? ($\nparallel / \parallel$)

   So, $(l_2, l_3) \in R \Rightarrow (l_3, l_2)$ ... R ($\notin / \in$)

   Similarly, $l_3 \parallel l_4$. Is l_4 ... l_3? ($\nparallel / \parallel$)

   So, $(l_3, l_4) \in R \Rightarrow (l_4, l_3)$ ... R ($\notin / \in$)

   and $(l_5, l_8) \in R \Rightarrow (l_8, l_5)$ ... R ($\notin / \in$)

∴ The relation R ... symmetric relation (is/is not)

3. In Fig. 2, observe that $l_2 \parallel l_3$ and $l_3 \parallel l_4$. Is l_2 ... l_4 ? ($\parallel / \parallel$)

   So, $(l_2, l_3) \in R$ and $(l_3, l_4) \in R \Rightarrow (l_2, l_4)$ ... R ($\in / \notin$)

   Similarly, $l_3 \parallel l_4$ and $l_4 \parallel l_2$. Is l_3 ... l_2 ? ($\nparallel / \parallel$)

   So, $(l_3, l_4) \in R, (l_4, l_2) \in R \Rightarrow (l_3, l_2)$ ... R ($\in , \notin$)

Thus, the relation R ... transitive relation (is/is not)

Hence, the relation R is reflexive, symmetric and transitive. So, R is an equivalence relation.

Application

This activity is useful in understanding the concept of an equivalence relation.

Note

This activity can be repeated by taking some more wires in different positions.

Objective

To draw the graph of $\sin^{-1} x$, using the graph of $\sin x$ and demonstrate the concept of mirror reflection (about the line $y = x$).

Material Required

Cardboard, white chart paper, ruler, coloured pens, adhesive, pencil, eraser, cutter, nails and thin wires.

Method of Construction

1. Take a cardboard of suitable dimensions, say, 30 cm × 30 cm.
2. On the cardboard, paste a white chart paper of size 25 cm × 25 cm (say).
3. On the paper, draw two lines, perpendicular to each other and name them X′OX and YOY′ as rectangular axes [see Fig. 5].

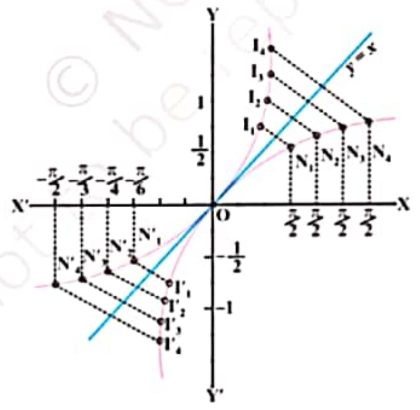

Fig. 5

4. Graduate the axes approximately as shown in Fig. 5.1 by taking unit on X-axis = 1.25 times the unit of Y-axis.
5. Mark approximately the points

   $\left(\frac{\pi}{6}, \sin\frac{\pi}{6}\right), \left(\frac{\pi}{4}, \sin\frac{\pi}{4}\right), \ldots, \left(\frac{\pi}{2}, \sin\frac{\pi}{2}\right)$ in the coordinate plane and at each point fix a nail.
6. Repeat the above process on the other side of the x-axis, marking the points $\left(\frac{-\pi}{6}, \sin\frac{-\pi}{6}\right), \left(\frac{-\pi}{4}, \sin\frac{-\pi}{4}\right), \ldots, \left(\frac{-\pi}{2}, \sin\frac{-\pi}{2}\right)$ approximately and fix nails on these points as N_1', N_2', N_3', N_4'. Also fix a nail at O.
7. Join the nails with the help of a tight wire on both sides of x-axis to get the graph of $\sin x$ from $\frac{-\pi}{2}$ to $\frac{\pi}{2}$.
8. Draw the graph of the line $y = x$ (by plotting the points (1,1), (2, 2), (3, 3), ... etc. and fixing a wire on these points).
9. From the nails N_1, N_2, N_3, N_4, draw perpendicular on the line $y = x$ and produce these lines such that length of perpendicular on both sides of the line $y = x$ are equal. At these points fix nails, I_1, I_2, I_3, I_4.
10. Repeat the above activity on the other side of X- axis and fix nails at I_1', I_2', I_3', I_4'.
11. Join the nails on both sides of the line $y = x$ by a tight wire that will show the graph of $y = \sin^{-1} x$.

Demonstration

Put a mirror on the line $y = x$. The image of the graph of $\sin x$ in the mirror will represent the graph of $\sin^{-1} x$ showing that $\sin^{-1} x$ is mirror reflection of $\sin x$ and vice versa.

Observation

The image of point N_1 in the mirror (the line $y = x$) is ________.

The image of point N_2 in the mirror (the line $y = x$) is ________.

The image of point N_3 in the mirror (the line $y = x$) is ________

The image of point N_4 in the mirror (the line $y = x$) is ________.

The image of point N_1' in the mirror (the line $y = x$) is ________.

The image point of N_2' in the mirror (the line $y = x$) is ________

The image point of N_3' in the mirror (the line $y = x$) is ________.

The image point of N_4' in the mirror (the line $y = x$) is ________

The image of the graph of six x in $y = x$ is the graph of ________ image of the graph of $\sin^{-1}x$ in $y = x$ is the graph of ________.

Application

Similar activity can be performed for drawing the graphs of $\cos^{-1}x$, t